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Teaching Geometry for the Purpose of Developing Ability to Do Logical Thinking

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THE development of ability to do logical thinking has long been counted one of the important objectives in the teaching of demonstrative geometry. In the past it was generally believed that the student acquired from the logic of geometry a mode of thought which he would use in solving many problems of life. Little or no direct attention, however, was given to the mode of thought itself, nor was any consideration given to the manner in which this mode of thought would be expected to function in various types of situations that are non-geometric. We have now come to believe that if transfer of this kind is desired, definite provisions must be made to teach for it. The teacher must definitely hold this outcome clearly in view and so arrange the teaching program that this outcome may be realized. It is possible that our whole teaching procedure needs to be reorganized.

If we hope in the teaching of demonstrative geometry to develop a type of thinking which is to function in all sorts of situations, then it would seem that we should emphasize logical thought as a thing of importance in itself and the pupil should be provided with abundant experience in applying this type of thinking in a great many different kinds of situations.

With the above considerations in mind the following plan has been developed and is being used in teaching geometry in the training school at the University of Kansas. Parenthetically, may it be stated that the writer is fully aware that the method is not entirely original? On the other hand, its use is not widespread and it would seem that something may be gained if workers who are experimenting with this method and with similar methods will exchange the results of their experiences.

It is planned to acquaint the pupils early with the nature of logical argument. This is done quite apart from any geometric setting. An attempt is made to get the pupils to see how some statements must follow as necessary consequences of certain others. In the beginning, there are various kinds of exercises involving simple syllogisms. Pupils are asked to tell what third statement we are forced to accept if we agree to accept the first two in such examples as the following:

1) All members of the drum corps are at least six feet tall.
2) Henry Williams is a member of the drum corps.

1) All Chinese are yellow.
2) Wing Toy is Chinese.

They are also asked to decide, in examples

of the following type, whether or not the conclusions reached are valid ones, that is, whether or not the third statement in each case follows as a necessary consequence of the first two:

1) Five hundred foreigners attended the meeting.
2) Peter Lagursky attended the meeting.
3) Therefore Peter Lagursky is a foreigner.

1) Only war veterans were in the parade.
2) William Smith was in the parade.
3) Therefore William Smith is a war veteran.

1) All hot days are disagreeable.
2) Wednesday was a disagreeable day.
3) Therefore Wednesday was a hot day.

Wherever it is possible, we use topics of current interest in these discussions, and pupils are encouraged to bring statements into the class for discussion.

Such examples with syllogisms as those just mentioned appear to be in rather general use. Not so much attention however, seems to have been given to detecting implicit assumptions in statements of the following type (enthymemes):

This is grand opera; therefore it must be good music.

The pupils are asked to point out what assumption, not stated, must be used in order to arrive at the conclusion contained in this statement. Other statements of the same kind are:

This substance is lighter than wood; therefore it will surely float in water.

A larger per cent of children of elementary school age attend school in the United States than is the case in any other country of the world. Therefore the United States has the best elementary school system in the world.

Throughout this part of the work the pupil should be approaching the point where he will regard the proof of a statement as consisting in showing that it follows as a necessary consequence of other statements previously accepted. He should also be developing ability to build arguments of his own and to analyze critically the arguments of others.

In order that the pupils may understand how a science is built upon a list of assumptions, the members of the class together build a miniature system, starting with a list of six or eight assumptions concerning a manufacturing plant. The following example will illustrate this point:

Suppose we assume the following statements to be true:

1) All employees of the White Steel Company are members of the Allied Workers Union.
2) No company can employ a member of the Allied Workers Union at wages of less than five dollars per day.
3) No foreigner can be a member of the Allied Workers Union.
4) William Smith is an employee of the White Steel Company.
5) William Smith and his brother, John Smith, together earn nine dollars per day wages.

Make as complete a list as you can of other statements that follow as a result of these five statements.

An attempt is made to get the pupils to understand the role that definitions and assumptions play in a science and in our thinking. They should see how a change in the assumptions of an argument will produce a change in the conclusion. They should also understand that if they are unwilling to accept the conclusion of an argument, they should either be able to find a flaw in the proof or they should be able to point out definitely which of the assumptions they are unwilling to accept.

The work described up to this point usually occupies about two weeks. The attention of the class is then directed to an informal consideration of geometric concepts. It is explained to the class that we plan to make a list of assumptions concerning the simplest of these geometric figures and to build a system upon this list of assumptions. Pupils are asked to bring to class statements which they are willing to take for granted. No limit is placed upon the number of such statements. Later in the course we may return to these assumptions and show that some of them can be proved from others, but

right here we accept all of the statements that the entire class is willing to take for granted. There is a considerable amount of discussion of these statements. As soon as one is seriously questioned we attempt to test it on the basis of other statements already accepted. From time to time we find it convenient to make additional assumptions. They are explicitly stated when needed.

We go through the course without the use of a textbook. An attempt is made to lead the pupil to discover for himself relationships that appear to be true, to formulate these into exact statements, and then to attempt to prove or disprove these statements. Thus the method is essentially one of formulating hypotheses and then critically examining them, rather than one in which the pupil defends statements presented to him in the textbook.

A great deal of careful directing is necessary on the part of the teacher to prevent much time from being wasted and still to insure that the pupil feels that he is playing a part in discovering these relationships. Some writers have advocated the use of the laboratory method for this purpose. This method is doubtless of great value and should be put to large use. However, its exclusive use would be likely to make the course one-sided. Among the methods which we use in an effort to find relationships to be proved are the following:

1) Experimentation with ruler and compasses.
2) Examination of converses of statements already proved or assumed.
3) The principle of duality. (No formal treatment of the subject of duality is contemplated here. However, the pupil should see the dual relationship between point and line as exemplified in the relationship between angles and sides of a triangle. He should see that by interchanging these two words in a statement a new statement can be derived which frequently expresses a relationship correctly.)
4) A study of locus problems.
5) A consideration of what takes place when a figure undergoes a continuous change.

The amount of subject matter to be covered in the course was not the chief consideration in planning this method. In common with Fawcett,* we regard the development of ability to think logically in all kinds of situations as being more important. It has been our experience, however, that we cover more ground in using this method than we had been able to cover previously.

At the conclusion of the year's work, the pupils ought to have a clear notion of what constitutes proof and they ought to be able to see the system as a whole, because they will have worked together to build it. They should see that what they have is a body of statements which are logically dependent upon a definite group of assumptions. During the year they have studied the essential characteristics of the methods used in establishing proofs (the analytic method, the indirect method, etc.), and they have had practice in applying these methods in non-geometric situations found in everyday life. It is hoped that with this method of teaching, the quality of the pupils' thinking may be generally improved, and also that this may be done without losing sight of the other objectives of the course.

* Fawcett, H. P., "Teaching for Transfer," *The Mathematics Teachers:* XXVII (Dec. 1935), 465–72.

Let's Check the Hypothesis

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It is quite generally recognized that, while the world needs mathematics more and more, the schools teach it less and less. The principal reasons offered to explain the loss of popularity of mathematics as a school subject are that it is badly taught and that the teaching materials are poorly selected. The chief values claimed for mathematics by its supporters are its utility and its preeminence as a mode of thought.

If it is true that the study of mathematics trains one in the art of logical reasoning, then our teachers of mathematics, because of their years of study, should be able to supply a very superior brand of thinking. And, if we allow that premise, these teachers ought to be able to arrive at sound conclusions regarding the matter which we mentioned first; namely, why a subject which is more needed is less revered. And it would seem that, if the suggested remedies are sound, their application would check the illness at least, if not give a complete cure. Unfortunately, this does not seem to be the case. Need we mention that, in spite of the careful analyses that have been made, in spite of our drastic curriculum revision, in spite of the help of the depression in purging our ranks of the weaker teachers, still conditions show no sign of improvement.

Is it possible that all our close reasoning has been wrong? Perhaps not; possibly we have been assuming an incorrect hypothesis. Let us examine the matter in some detail. We have assumed that it is valuable to be able to reason logically. Teachers point with especial pride to geometry as a subject which teaches children to reason logically; but geometry is in the van of unpopular subjects. This fact furnishes us a clue. Since we admire logical reasoning, we presume that every one does likewise. With great reluctance, the opinion grows that admirers of logical reasoning constitute a feeble minority of our population. Along with reverence for the classics, the habit of thrift, belief in paying one's debts, pride in earning an independent living and providing for one's old age, logical reasoning has apparently become one of those sterling virtues of antiquity which is outmoded, old fashioned, obsolete, and out of place in this bright new world.

And then, we often speak of the beauty of truth as portrayed by mathematics, just as though truth were something that people admired as a matter of course and sought more of. How often do we quote, "As sure as two and two make four." But do we mistakenly imagine that most people like to contemplate this incontrovertible fact? Isn't it open to question whether most of us like to face facts? With hardly enough money to pay the first instalment and no more in sight, who is grateful for exact information as to the total cost of the contemplated purchase? Who likes to have his dreams of future grandeur deflated by cold facts? Who is willing to let mere facts interfere with profitable theories?

If we start with a new hypothesis we reach a new conclusion. Suppose that our students and their parents look about them and observe the success of the propagandist—he never quite lies but he manages to deceive by telling half-truths; the high-power advertiser—he preys on people's fears and weaknesses; the high-pressure salesman—he sells most to those who least need and can least well afford his wares. Contrasted with the material possessions of these successful men are the poverty and obscurity of those who devote their lives to serving as votaries at the shrine of pure science. Our students may well conclude that the men who receive

the richest rewards of wealth and prominence have the most comfortable philosophy of life. They may decide that the man who is thrifty is a fool, denying himself so that he can give up his savings in the form of taxes; that the government will take care of anyone who doesn't want to take care of himself; that to earn an honest living is foolish; that an ounce of pull is worth a pound of push; that virtue is its own slim reward, but the chiseler is smart enough to get his fun now; that the only disgrace about helping oneself to some other person's earnings is getting caught doing it; that it is smart to violate the spirit of the law while keeping within the letter of it; that when truth is in opposition to emotional belief it is not desirable; that courses of action are most profitable when decided on the basis of expedience; and, that if you take advantage of every opportunity to benefit yourself at the expense of the rights of others and guard against contributing anything to help society, you will be greatly admired by your associates.

One more point to complete our new hypothesis. The point has already been implied but needs specific statement and amplification. To obtain a knowledge of mathematics requires continuous effort and hard work; continuous effort because the structure of mathematics is like that of a lofty building which is constructed with each section resting on the sections below and supporting the sections above. If it were a system of one-story bungalows, the failure of one unit would not have such a devastating effect on the entire edifice. Hard work of the kind required by mathematics was once considered respectable. The dignity of toil has been extolled in poetry and song. Now few find joy in work itself; it has value only for the reward it promises and, even then, is not alluring unless the pay is large and immediately available.

Accepting the hypothesis that our attitude toward life has changed in the ways indicated, it is easy to understand why mathematics is rapidly disappearing from our schools. Beyond a little very simple arithmetic, mathematics has little to offer that is generally needed. In fact, most of it is definitely not wanted because it runs counter to our modern philosophy of life. If the assumption is in agreement with the facts of the case, one is drawn toward the conclusion that, until the trend of our social consciousness is changed, mathematics will decline to the point where it will be studied only by those who need it to prepare for a trade or profession.

If the above estimate of the situation be accepted, it would seem that any hope that the grand old science of mathematics will retain or regain its position in the popular esteem must rest, not on arguments in its behalf or on attempts to jazz it up to date, but upon a confidence that the public mind will eventually return to a sounder philosophy.

Suggestions for Teachers of Mathematics

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Importance of the Teacher

The teacher is the most important factor in our scheme of education. James A. Garfield is reported to have said that Mark Hopkins on one end of a log and a boy on the other end made a school. Fine buildings and fine equipment are splendid helps but without the inspired and enthusiastic teacher they are nothing. There is no substitute for good teaching. Even lazy and indifferent pupils are oftentimes roused out of their indifference and lethargy by a real teacher. In his hands the seemingly dry bones of the subject will glow and take on the forms of life.

Motivate the Subject

Many pupils do not appreciate the great value of algebra and geometry and so do not give the proper attention to these subjects. The pupils are told continually by outsiders that Mathematics is "bunk" and of no practical account anywhere at any time. Teaching will be easier and learning will proceed more rapidly if this prejudice is broken down. Also it is the pupil's right to know why he is required to study any certain subject. Nearly every pupil will be more interested in a subject if he is shown that it is vitally connected with his daily life and work. I feel sure that if your pupils fully understand that mathematics is just plain common sense and that it occupies such an important place in nature and in all of the world's activities they will take more interest in the study and they will achieve better results.

Give an Interesting Presentation

Even if the subject is uninteresting to a pupil an interesting lively presentation by an enthusiastic, wide awake teacher often times arouses the pupil and the bias and prejudice are overcome. How dull and dry and stupid and lifeless were the hours spent with certain teachers in our own school experience. We also remember with pleasure and appreciation those teachers who inspired us by their knowledge and enthusiasm. We owe it to ourselves to pass on this inspiration to our own pupils. How will they remember us?

In Algebra Teach Principles

In each topic there are certain principles that govern the solution of problems. If a pupil fails with a problem it is a pretty sure guess that he has not mastered the underlying principles. Have him review these principles and the difficulty is removed.

In Geometry Emphasize Analysis

If the student does not acquire from his study of geometry some power of analysis, then he has not benefited much from the study. Of course, there are some by-products like neatness, accuracy, precision of language, etc., but the main result should be analytic power. He should be able to attack any problem or life situation, look into its possibilities and arrive at a conclusion more efficiently than before he studied geometry. The facts—most of them—will soon fade out of his memory, but the method of thought, if once acquired, will become a part of the pupil's mental equipment and continue with him through the years.

Illustration and Application

We as teachers and students are able to appreciate the science for itself. We are able to see and admire the beautiful logical structure of geometry and the powerful generalizations of algebra. Not so with the high school boys and girls. Their immature minds cannot grasp the logic and beauty of the Pythagorean Theorem.

Show them how it will measure some unknown distance or how the plumber and carpenter use it daily in their work. Then the theorem will lose its abstraction and have some meaning in the pupil's life. Illustrate the wonders of our Number System. Contrast it with the Greek and Roman System. Show the students that algebra is the language of science; how all of science and industry depend upon the principles and processes of algebra. Concrete illustrations and applications greatly facilitate the learning and memory of the abstract principles.

Teach the Pupils How to Study

Subject Matter is easily forgotten; in fact, much of it should be forgotten. Habits of study, however, become engrained in one's mental nature and become a part of one's mental equipment. Irregular and fitful habits of study are the cause of much of the failure. We have not done our full duty unless we help our pupils to improve their habits of study.

Recitation Period

Use a part of every period for supervised study of the new lesson. It is tiresome, to good students especially, to go over problems that they have already mastered. Use a few minutes for checking errors and answering questions. Then interest and stimulate the pupils with some new problem or application. That plan, I believe, will make the recitation period more interesting and profitable to the students.

Expression by the Pupil

Talk as little as possible during the recitation period. We as teachers are prone to talk too much, either about the lesson or about some totally irrelevant subject. This, it seems to me, is a mistake and a waste of time for the pupils. We should get them to talk. Nothing does the student as much good as expression. If a question is asked have some pupil answer even if he consumes more time than the teacher would. Discussion and expression will give the pupil valuable practice in the use of technical language and will clarify his ideas. If he cannot give any discussion more study is needed. We teachers should talk little, the pupils much.

Attitude Toward the Pupils

Let us give praise to the pupils, not censure. Praise will spur them to higher endeavor; censure will stifle their efforts. Praise will give life and interest; censure will deaden and kill. The pupil needs friendly, constructive criticism to guide him along the way.

Keep an Open Mind

Teachers naturally are conservative. So one of our most common faults is "getting into a rut." A rut and a grave are alike except in the dimensions. We do our teaching in the same way year after year, never thinking that any change at all would be beneficial. We are very slow to accept new ideas and plans. Of course, just because an idea is new does not make it good. By the same token, an idea is not good just because it is old. The alert, progressive teacher should ever be ready to test and try new methods, and to use whatever of good they may contain. The old adage—"Be not the first by whom the new is tried nor yet the last to lay the old aside"—is pretty sound advice.

Review Frequently

One study of a topic or unit is not sufficient for mastery. Frequent reviews are needed to clarify and extend the principles previously studied.

Teach and Test

The teaching of a topic should be immediately followed by a test. This will show whether or not the pupil has mastered the principles and also whether or not the teacher has presented the material correctly and clearly. If a large number show failure then the teaching must be repeated. Then test again. This process should be repeated until all have mastered the topic. Short tests containing only two

or three problems are sufficient for this purpose.

Learning Should Be Intensive

Superficial learning has very little value. It soon fades from the memory. Neither pleasure nor profit can follow from such study. The pupils do not appreciate this fact and do not often study to the point of mastery. They need the guidance of the teacher. Thorough mastery of a few principles is better than the superficial study of many.

Kindness and Sympathy

A kind and sympathetic attitude toward the pupils always produces the best results. Even though the pupil deserves harsh treatment a firm and kindly method will always secure the best response from him. *Sarcasm can never be justified.* Its use embitters the pupil and lays the teacher open to criticism. Never use it.

Professional Reading and Study

As soon as the college course is completed and a position secured many teachers cease to study. New developments in subject matter and changes in methods of teaching are unknown to them. They teach as they were taught and do not dream of progress. Of course, this is a mistake. No teacher can hope to do his best work or to properly develop his powers without continual study, both of subject matter and teaching methods. Professional journals and books should occupy an important part of a teacher's reading. Not that he should adopt every device and method offered. Rather, he should carefully evaluate each one in the light of his own experience and adopt only those that are sane and usable. The progressive teacher will attend the Institutes and Conferences and Teacher's Meetings not as a fanatic but for the purpose of learning and of helping to bring out by means of research and discussion the best possible plans and methods. In short he will always be found doing every thing in his power to improve the character of his work and to make the teaching of mathematics more vital and interesting.

Mathematics in Relation to Physics

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MATHEMATICS is undoubtedly applied as extensively in physics as in any other physical science. However, in this discussion, one must not lose sight of the fact that what is said regarding the relationship between mathematics and physics applies equally well to many phases of other physical sciences such as astronomy and various fields of engineering.

Long before the birth of science as we know it today, the Greeks, in search of a certain beauty in nature, evolved our first mathematical processes. To the Greeks mathematics was similar to a game in that at the outset certain rules and regulations were established. These rules combined with the qualities of symmetry and orderliness acted as a guide to the mathematicians as they developed the mathematical processes of algebra, trigonometry, calculus, and other more advanced phases of the subject.

The symmetry of numbers and geometrical figures, as illustrated in the study of algebra, geometry, and trigonometry, was well understood long before any extensive applications were found. It seems almost a coincidence that the Greeks should develop a mathematics which could be used in physical science with so little modification. As a result, the whole subject of forces at rest or statics developed with amazing speed, since the direct correlation of forces as vectors could be made immediately with the subject of trigonometry. At the outset, the relation between mathematics and physics, in those problems involving algebra and trigonometry, is such that mathematics was formulated first, and in the development of physical science which followed it was found that a one-to-one relation existed between mathematics and physical science. Mathematics developed the tool which the physicist used to express the laws which he found.

On the other hand, sometimes the discoveries of physical sciences are such that a branch of mathematics is required which has not yet been developed. Now the scientist points the way and the mathematician develops the tool necessary for the work. This was the situation at the beginning of the seventeenth century when Galileo discovered experimentally the relation between the time and the distance passed over by a body having a constant acceleration. That was the first occasion in which time was considered a continuously changing quantity and in which any degree of accuracy was obtained in the measurement of time intervals. The idea of varying velocity led Newton to think in terms of a fluxion or a continuous change as he attempted to define instantaneously the changing quantities of distance and time. Leibnitz (1646–1716), a German mathematician, developed the idea of fluxions into the present day calculus, a mathematical tool indispensable in solving many physical problems. Thus we find that physical science often aids in opening up for the mathematician new frontiers of knowledge.

The study of nature leads to weighing and measuring. This in turn leads to the establishing of relations which can be expressed in mathematical form and hence studied by mathematical methods. In the large majority of instances these measurements are indirect, that is, some related quantity is measured directly and the value desired is computed from it. Calculations of this sort bring into play a considerable amount of algebraic and geometric theory. Thus, mathematics in the form of arithmetic, algebra, and geometry, along with the application of the analytical method, furnishes the peculiar study that gives to us the command of nature in its quantitative aspect.

Nature is so completely mathematical that some of the more exact natural sciences such as physics and astronomy are, in their theoretical phases, largely mathematical in character. Other sciences, which until now have been compelled by the complexity of their phenomena and the inexactitude of their data to remain descriptive and empirical, are developing toward the mathematical ideal. Research workers in these latter fields are proceeding upon the fundamental assumption that mathematical relations exist between the forces and the phenomena, and that nothing short of the discovery and formulation of these relations would constitute definite knowledge of the subject. Progress is measured by the percentage of the basic principles that will yield to this treatment.

Although mathematics is a tool for the physical sciences, one often finds it a very inefficient tool in the hands of students. A complaint very common among physics teachers is that the students are not able to apply in physics the mathematics they have previously learned. Other teachers, not only in this country but also in England, report a similar weakness in their students. Mathematical feebleness and fallibility are the birthright of a large percentage of every class in beginning physics. This is not to be wondered at since no mathematics has been taken up for a year or more previous to their work in physics. With their attention absorbed by the difficulties of the new work in physics, it would be a marvel if the students were to recall the needed formula from the algebra of their first year or the propositions from the geometry of their second year in high school.

The best solution so far suggested for the above problem seems to be to develop mathematics and physics simultaneously, to the mutual advantage of both. The physical genesis of a mathematical problem lends interest and life to it while the application to physics tends to fix the mathematical result more firmly in mind. This suggestion has not been tested to any great extent in our country, but in France and Germany mathematics and physics have been taught side by side for years with excellent results; there does not appear to be any immediate prospect of a change in the peculiar association of these two subjects.

Of the various branches of mathematics, algebra finds the most extensive application in a beginning course in physics. Of the various topics in algebra, the students are found to be especially weak in interpreting formulas and equations, in passing from literal to numerical expressions, in applying variation, and in ratio and proportion. This should give the teacher of mathematics the cue to pay special attention to these topics. The last three of the above topics really fall under fractions and should be treated in a simple, straightforward manner, without the use of such words as means and extremes, antecedents and consequents, and rules involving these terms. The plain theory of fractions is all that is needed. All of the above topics lend themselves well to concrete treatment and can be made some of the most interesting materials of the whole subject. At the same time, studying these topics should prepare our young people to meet the justifiable expectations of the physics teacher, that students coming to them be readily able to answer such questions as this: If $T=2\pi\sqrt{l/g}$, the formula for the period of a pendulum, how does T vary with l? with g?

It would be a wise procedure if the algebra teacher would ascertain from the physics teacher in the same school quite specifically what mathematical relations are to be discussed and what mathematical problems are to be solved in the physics course together with the notation in which they are to be expressed. These relations could be taken up in the same notation in the algebra work. There is no reason why the equation $S=\frac{1}{2}gt^2$, the formula for the distance covered by a freely falling body, or $pv=c$, Boyle's law,

should not be treated in algebra as well as $y=ax^2$ or $xy=c$. If the student learns in algebra that an equation may be solved for S, or for t, or even for g, he will not, as some students actually do, first replace S, t, or g by x, and then solve for x. The recognition of the possibility of expressing any quantity that occurs in any equation in terms of the others is highly important and should be clearly grasped by the student.

In the field of secondary education the relation of mathematics to physics is that of a set of tools. The equation is by far the most valuable tool. Elementary physics courses contain perhaps 150 or more simple equations of the type $A+B=C^n/D$ (where $n=1$, 2, or 3). The physics student must substitute numbers for these letters or be able to solve for any one of these letters. It is indeed discouraging to the physics teacher when he finds that often as high as 50% of his students at the beginning of his course have tremendous difficulty in doing this task or are unable to do it at all. To overcome this handicap, the physics teacher must of necessity take time out of the regular lecture period to review certain phases of elementary mathematics before he can proceed. A closer correlation between the subjects of mathematics and physics throughout the time they are being presented to the student seems to be the most promising solution to this vital problem.

Out of the Past

A PLAY, *in two acts, created, and presented in a school assembly, by a ninth grade class, under the direction of* FLORENCE BROOKS MILLER, *Shaker Junior High School, Shaker Heights, Ohio.*

As opportunities presented themselves, historical bits of information were injected into the program of a ninth grade algebra class. Among these were the development of our number system, and the use of mechanical devices, from very early times up to the present, to aid man in keeping track of things as well as to calculate.

It was suggested one day that a dramatization of such information would be interesting and might be used in a school assembly. Several plans were discussed and when one was finally decided upon, the whole class felt that it was a creation of the group. This play is not the work of one person. Suggestions, criticisms, changes, by members of the class, made it what it is. Every member of the class was cast in the play when it was enacted before the school.

The stage set for the second act was rather unusual. The dramatic director of the Shaker Heights Schools, E. Benson Sargent, assisted us immeasurably. What appeared to be a large circular window, with a diameter of ten feet, was in the background. Scrim was stretched across it, making of it a screen upon which pictures were thrown from a stereopticon. The use of lighting made it impossible to see anything back of the screen until the right time for the various events mentioned by the scholar, to appear in the form of a tableau or pantomime. Dimming the table light, which shone upon the scholar and his visitors, the pictures were made visible on the screen. (On plain glass slides pictures were painted which depicted the eight scenes which were appropriate for the tableaux.) Bringing up a bright light back of the screen, the picture faded out and the tableau or pantomime was clearly seen.

OUT OF THE PAST

Act I

Time: The present

Place: A Chinese Restaurant

(A corner of the restaurant is seen. The Cashier's desk is near the door at the left. Two or three tables have people seated at them eating. At a table near the front of the stage are a young man and a lady who are getting ready to leave.

Young man (to waiter): Check please.

Waiter: Yes, sir. (Makes out the bill)

Lady: I enjoyed this very much, Frank.

Young man (helping her on with her coat): I did too, Esther. The food is certainly good here.

(They walk over to the cashier's desk where they see an abacus.)

Young lady: What is this?

Cashier: It is what I do all my calculating upon. It is called an abacus. Have you never seen one before Miss?

Young lady: No, never. I have heard of them. It reminds me of the beads on wires we used to have in the kindergarten.

Cashier: Yes? See, I use it this way. (Working the abacus very fast)

Young man: You get lots of practice I guess.

Cashier: Yes, I do. I learned in China. There these are used everywhere in banks, in counting-houses, stores and so on. Are you interested in such devices, Miss?

Lady: Indeed I am. I have always been curious about calculating machines.

Cashier: There is in this city an old Chinese scholar who has collected and made a study of all manner of instruments used in calculating and recording numbers. I shall be very glad to give you his address. Perhaps you have heard of him. I speak of the Honorable Doctor Kuan Chan.

Lady: No, I have never heard of him.

Young man: I have. Do you think he would give us an audience?

Cashier: Yes. Especially if he knows that you are interested in these things. He craves listeners!

(Writes the address of the Scholar for them.)

Young man: Thank you very much.

(To Esther) I shall phone and make an appointment and if the worthy doctor will see us we shall go there tomorrow.

(They go out and the curtain is drawn.)

Act II

Time: The following day.

Place: Home of the Honorable Doctor Kuan Chan.

(The old scholar is seated in an easy chair reading a large book. At his feet sits his small grandson. The door bell rings. The child goes to the door.)

Young man: Good afternoon, my young man. The honorable Dr. Kuan Chan is expecting us. Miss Grant, and Mr. Stapleton.

Boy: Oh! Come in. My honorable grandpapa is right in this room awaiting your arrival. (They enter.) Honorable Grandpapa. Here are guests. Miss Grant and Mr. Stapleton.

Scholar: Come right in. Lee, draw up comfortable chairs. Won't you lay off your wraps? I have been anticipating with pleasure your visit since our conversation over the phone, Mr. Stapleton. So this is the young lady who is also interested in mechanical devices used for calculation.

Lady: Yes, Dr. Kuan Chan. I am looking forward to hearing much of interest this afternoon.

Scholar: Well, there is much to tell and to show you. I have gathered some of the old relics from my collection and we shall start with these pebbles as they were probably used earlier than any other thing, as an aid in counting.

Lady: Pebbles!

Scholar: Yes. (Picture of a jungle with a clearing appears on the screen) Primitive man evidently counted on his fingers, as is very natural for us to do today. It is easy to keep track of a small number of things in that way. But when, say, twenty or any number over ten is counted, confusion might creep in, so pebbles or shells were used.

(Picture fades away and behind the screen is seen the acting out of the story told by the scholar)

Let us imagine a group of captives, taken by a primitive tribe. The head of the tribe wants to know how many captives there are. As each captive passes the chief, one of the tribe drops a pebble on the ground. When ten are so counted or in other words when he has counted all his fingers, the pebbles are gathered up and one stone is placed to one side to represent ten. This continues until a heap of ten pebbles each indicating ten is piled up. Then one is placed in another position to represent one hundred.

(Lights go off back of the screen and we no longer see the pantomime.)

Young man: Dr. Kuan Chan, I have heard that the Latin word for pebbles was calculi and that the name of a branch of mathematics the Calculus, came from it.

Scholar: That is more than likely.

Lady: Wouldn't our verb "to calculate" come from "calculi" too? Just think how much we owe to the pebble!

Scholar: You are right, Miss Grant. (Turning to his grandson) Lee, bring me the knotted cords.

Boy: Here they are Grandpapa. (Hands him the knotted cords.)

Scholar: Knots on cords have a long and varied history. A Chinese philosopher in the sixth century B.C. referring to the earlier use of the knotted cords said, "Let the people return to knotted cords and use them." In 1872 in taking the census for India, the Santals in the wilder parts of India used knots on four colors of cords, the black for the adult men, the red for the women, the yellow for the girls and the white for the boys.

Young man: I have read that the Zuni

Indians had a system of knot numerals. A medium knot indicated 5, and this with a small knot before it indicated $5-1$, whereas if the knot came after the medium sized one, the number was $5+1$. A large knot indicated 10, and a small knot was used either before or after it to indicate 9 or 11 respectively.

Scholar: Yes, see I have a similar kind here. (Showing such a grouping of knots)

Lady: That reminds me of the Roman Numerals. If I is placed before V it stands for four, and if after V it means six.

Scholar: Yes, Miss Grant, it is the same idea. The knotted cords reached their more elaborate forms among the Peruvians. The different colors, the sizes of the knots, the distance between the knots, all had some significance.

Young man: The surveyor's chain is a modern example of the knotted cords, isn't it?

Scholar: Yes and the knotted cords are found in various forms of religious regalia and most pronounced in the rosary upon which prayers are counted by Catholic Christians.

(A scene showing the bridge over the Ister River is seen on the screen as the Scholar continues.)

The famous historian, Herodotus, who lived in the fifth-century B.C. tells us that the King of Persia handed the Ionians a thong with 60 knots on it, as a calendar for two months. He also tells us that Darius, who also lived in the fifth century B.C. bade the Ionians to guard the floating bridge that spanned the Ister.

(Picture fades away and the acting back of the screen is seen as the Scholar tells the story.)

He tied sixty knots in a thong, saying, "Men of Ionia, do keep this thong and do as I say. So soon as ye shall have seen me go forward against the Scythians, from that time begin and untie a knot each day; and if within this time I am not here, and ye find that the days marked by the knots have passed by, then sail away to your own lands.

(Lights go off back of the screen and we no longer see the acting.)

Lady: How very interesting!

Young man: And what are these sticks? (Lee brings them over)

Scholar: Those are tally sticks. You see they are in pairs. (They examine the tally sticks.)

Lady: They have notches.

Scholar: The idea of keeping records on a stick is very ancient. On a bas relief on the Temple of Seti, 1350 B.C., at Abydos, Thot is represented as indicating by means of notches on a long frond of palm the duration of the reign of Pharaoh as decreed by the gods.

Lady: 1350 B.C.!

Scholar: In the Middle Ages the tally formed the standard means of keeping accounts. The notches were cut before it was split so as to allow each party to have the same record, whence the expression "our accounts tally."

Lady (examining one closely): The twentieth notch is larger and deeper than the others.

Scholar: Yes and from that fact we get the name "score" for twenty.

Lady: That's right, score means cut, doesn't it? Because the twentieth cut or score is the largest it is called the score. How interesting!

(On the screen is shown a bank interior in England, in early times.)

Scholar: If a man lent money to someone, the amount was cut on a tally stick by notches. Identification marking and the date were also cut on it. This was then split, the borrower keeping the foil and the lender the stock. That is the origin of the name "stock" in "stockholder" of a company.

Young man: Is that where the term "bank stock" comes from? I always wondered about that.

(The acting back of the screen shows men entering the bank handing over money and getting tally sticks.)

Scholar: The tally was used all over

Europe as early as the 13th and 14th centuries.

Lady: Though the tally sticks are not common today, expressions derived from them are. We speak of the score of a game, a Bridge score pad, and accounts tallying.

Young man: Yes. And don't forget the Bank stock.

Lady: Some of us today would like to forget that we ever had bank stock! (They all laugh.)

Young man: What about that board over there? What was it used for?

Scholar: Well, that comes next. Bring it here, Lee. An early method of writing down numbers in a systematic way was the use of the dust board. Parallel lines were drawn by the finger in the dust or sand strewn on the board. Upon the lines, pebbles were laid. When the tenth pebble was about to be placed, it was laid on the second line and the nine already placed were removed. Continuing to count, pebbles were again placed upon the first line, the tenth one being placed on the second line with the first ten.

Lady: Writing paper was unknown then was it not?

Scholar: Yes. Papyrus was used as writing material about 2000 B.C. and was used in Italy in the 12th century. For centuries all writing materials were very expensive.

Lady: It was a common custom to write upon sand wasn't it in early days? We read in the Bible how Jesus wrote on the sand.

(While this conversation is going on there is a scene suggesting Palestine and the acting back of the screen will then be seen. An old mathematician studying a geometric figure drawn on the sand.)

Young man: I have seen pictures of ancient mathematicians, Archimedes, for example, studying a geometric figure drawn on the sand of a shore.

Scholar: Yes. The dust or sand give the counting board and other forms developed from it the name "abacus." An old Semitic word, abaq, means sand.

Lady: That is what the cashier in the restaurant called his calculating frame yesterday, an abacus.

Scholar: The dust abacus finally gave place to a ruled table upon which disks or counters took the place of pebbles. It is as interesting as a game, to add, substract, multiply, and divide on the counting board. Spaces as well as lines had a distinct decimal value. There could not be more than four disks on a line, nor more than one in a space. If there should be five on a line one is carried to the next space in their stead.

Lady: Do you suppose that is why we use the expression "carry" in addition?

Scholar: Yes. And in subtraction, using the counting board, disks are actually borrowed when needed.

Young man: Those crosses marking every third line are only to help in the reading of the number as do the commas which are used in writing our large numbers, I suppose?

Scholar: Exactly. This board was commonly called by merchants a counter.

Lady: Why! We call the long table over which goods are bought and sold in a store, a counter.

Scholar: Yes. We can easily understand how that name came into use now can't we?

(The scene on the screen shows a business street in old London, and the acting, seen back of the screen shows the counter used in a London Counting House.)

The English Exchequer got its name from the checkered board or counter.

Young man: Then our word "check" can be traced to this counting board too?

Young man: The abacus used by the Chinese Cashier is very much like the counting board. Instead of lines there are wires and instead of counters there are the beads strung on the wires.

(Scene on the screen, a street in China. Back of the screen is enacted a scene in a counting house in China where very rapid work is being done on an abacus.)

Scholar: This form of the abacus is still

used commonly in China, Japan, Russia and parts of Arabia. The computer works very rapidly as an expert typist or pianist and secures his results in adding, subtracting, multiplying, and dividing much more quickly than can be done by our common methods. (Scene closes)

When people began to write numbers without drawing lines to keep them in place the difficulty of position showed up and the zero came into use.

Lady: That gives me a new idea of the zero. It is a sort of stopper, isn't it?

Scholar: Yes. It keeps the digits in their right places.

Lady: Doesn't digit mean finger?

Scholar: Yes. We see again a trace of counting on fingers, don't we? The blackboard and chalk, slate and slate-pencil, paper and pencil, or pen have taken the place of the counting board and like forms.

(Scene on the screen—an old-time school room. This fades away and back of the screen is enacted a scene in an old time school room with pupils working at the black-board, on slates, etc. The teacher is showing a class how to use Napier's rods as the Scholar is telling about them)

Young man: What are those sticks, Dr. Kuan Chan? (Lee brings them over.)

Scholar: These sticks are called Napier's rods. John Napier, a Scotchman, who lived in the 16th century invented this device for multiplying any two numbers together without having to know the multiplication combinations. By placing the sticks so that the multiplicand is seen along the top, so, and the multiplier is seen on this index at the side, so, the product is found on this line beside the multiplier, here. It entails a little bit of adding but no multiplying.

Lady: How very clever! Let me see if I can do it. Frank, you write down what I say. (She places the sticks together and works a problem. Frank then multiplies the numbers our way and finds the answer to be correct.) Scene closes.

Young man: We use commonly so many shortcuts and time-saving devices in our problem solving, without realizing how much they really mean to us.

(On the screen is shown a modern business office with desks chairs, etc. Then as that fades out we see a very busy office with graphs on the walls, interest tables, also. Adding machines, typewriters, slide rule, mechanical calculus etc.)

Young man: When we think of the tables and graphs made out so that all we need do is to learn how to read them the comptometer that performs all manner of calculations for us when we touch the right buttons, the slide rule that aids us in estimating, computing and checking results, the Integraph that aids engineers in the solution of problems involving the Calculus, we are impressed with the inventiveness of man.

Lady: And also, how dependent upon the primitive devices, the most intricate modern inventions are. We are indebted to you Dr. Kuan Chan, for teaching us so much of the history of calculation.

Scholar: It has been a great pleasure to me to have this visit with you who have like interests with me. May I hope for future visits from you as we have touched only a few of the many historical developments dependent upon mathematics. (They put on their wraps)

Lady: We shall plan to come back soon Dr. Kuan Chan. Good-bye.

Young man: Good-bye, Dr. Kuan Chan.

(Curtain)

"The Mathematics Teacher" wishes all of its subscribers a Merry Christmas and a Happy New Year!

Problem Solving in Algebra

By D. McLeod, Daniel McIntyre C. I.
Winnipeg, Manitoba, Canada

No apology is needed for re-introducing in the pages of *The Mathematics Teacher* a topic which may be time-worn. The subject of problem solving is always in place and ever calls for solution.

The reason for the writing of the article which follows was a remark made recently by one of our bright students in his last year of high school. "Algebra would be easy," said he, "if there were no problems in it. They are my stumbling-block." One hesitates to think of the plight of a poor student in this respect!

Immediately the question arises: Is this condition universal? What is so difficult about problem-solving as compared with simplifying fractions, factoring, deriving formulas, or finding the root of an equation?

As teachers of mathematics, we must face the facts squarely. This is the situation. In tackling an algebraic problem the student is thrown at once on his own responsibility and ingenuity—if he has any. Before him is what appears to be a maze of words from which facts are to be selected and related and from which also an underlying idea of equality must be evolved to secure that equation which is necessary for the finding of the unknown. Cut-and-dry forms or stereotyped methods of approach such as form the starting points in the simplification of fractions are not present here. The pupil must select, translate, relate—all in one question. Frequently, prolonged effort results in failure. Problem solving, in other words, appears detached and new, with little or no relation to the work previously covered. The student is on "a peak in Darien" and needs the heart of "stout Cortez."

What are the main reasons for failure in this department of school work? Perhaps pupils have a mistaken idea about difficulties and are beaten before they begin.

1. Pupils in many cases do not understand the *language* of algebra. Terms such as sum, product, quotient, integer, excess over etc. have not been learned properly. There is a tendency among even good teachers to neglect definitions in the text. It is easy to get by without them. How many pupils actually *read* the algebra text? Recently a high school examination paper asked for the meaning of a determinant. The reason was probably to see whether that excellent method of solving linear simultaneous equations was being employed as a rule of thumb or as an intelligible thing. The prevailing answer to the above was, I learn, as follows: "It is a rule for solving equations." Algebra has a language of its own—symbols. One letter may signify a word, a phrase, or a sentence. The symbolism of algebra should be taught early and constantly. We are indebted to the authors of modern texts for the treatment given to this topic of algebraic representation just before the sections on problem solving. We refer to such statements as "5% more than x," "the greater of two numbers whose sum is x if the smaller is y" etc. A pupil surmounts his first barrier when he understands the language of algebra and the meaning of terms of common occurrence. Can he translate from English to Algebra or vice-versa? Consider the following problem: The excess of a number over 10 is twice as large as the number. Find it. We venture to say that a large number of students, fresh to the work, would regard "excess over" as a division process.

2. As problem solving in the final issue involves the formation and the solution of an *equation* the idea of equality present in a problem must be sifted out clearly from the facts presented. Here is the next difficulty. Memory work and certain tricks

used in other sections of the work must here give way to clear and perhaps original thinking. No hard and fast rule can be given beforehand as to that on which the idea of equality hinges. The underlying thoughts of equality are so varied. Each question, apparently, is taken on its own merits. After listing the facts and representing them algebraically, the pupil must ask what two things are equal. This is done by grouping the facts around the number represented as the unknown in such a way as to form an equation.

An attempt is made in several algebra texts to make problem solving easier by grouping questions under definite headings such as *Interest Problems*, *Time Problems*, *Mixture Problems* etc. Whether this is the right thing to do or not is questionable, but still it simplifies the work. There is this advantage to the student that it limits the number of the *equality ideas* necessary for the equations. For example, in interest problems we see that equations will depend on one or other of the following: $I_1+I_2=\text{total } I$ or $I_1-I_2=x\$\text{'s}$ etc. Similarly, in motion problems we have $D_1=D_2$ or $D_1+D_2=\text{total } D$ or $T_1-T_2=x$ hours etc. Then, again, in mixture problems, statements of the following kind repeat themselves: Value of cheap tea + value of dear tea = Value of mixture; or Salt in old solution = Salt in new (if water is added); or Water in old = Water in new (if salt is added). The ambitious pupil will sometimes master all the possible cases and make himself equal to the task of such problem solving. Even the poor pupil stands a chance he does not have if the problems are not *so* grouped.

3. The next difficulty is the actual establishing of the equation itself in correct algebraic form and content. Many teachers regard this step, if properly done, as worth 60% of the question. Often, after the idea of equality is clearly seen and the necessary translation and grouping of facts are made, the equation itself has some term or part of a term wrong. Common errors in this regard are: (1) Both members of the equation are not expressed in the same *unit*. Take the problem:

I go 75 miles into the country at a certain rate. I return at a rate 5 miles per hour faster than my rate going. I take 45 minutes less time on the return trip. What were my rates?

A common error is to write $\frac{75}{x}=\frac{75}{(x+5)}+45$ where $x\equiv$ the rate in miles per *hour* for the outward trip.

The same error may be made in a problem like the following:

I buy 100 stamps some at 2¢ each and the rest at 3¢ each. I pay $2.25 in all for them. How many of each kind did I buy?

Here the tendency is to state the equation as follows: $2x+3(100-x)=2.25$. This is done even though the left member is expressed in *cents*. The right member should, of course, be 225.

(2) Some implied fact is omitted. Here it is necessary to read between the lines. Consider a problem such as this:

A passenger train leaves a certain place 40 minutes after a freight train and, travelling at 30 m.p.h. overtakes the freight in 2 hours and 40 minutes. What was the rate of the freight train?

The equation obviously depends on the fact that Distance covered by freight = Distance covered by passenger train. The implied fact is that the *time* the freight has been travelling is 3 hours and 20 minutes. So, if x m.p.h. $\equiv$ rate of the freight

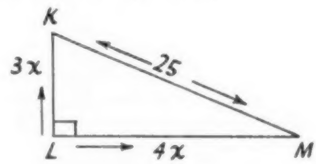
$$3\tfrac{1}{3}x=2\tfrac{2}{3}\times 30.$$

Though we are dealing with first degree equations in one unknown chiefly, here, we may incidentally note that in quadratic equations a common implied fact in the setting up of an equation is the geometric truth established in the Theorem of Pythagoras.

Example:

A and B start from the same place, A going East at 4 miles per hour and B North at 3 miles per hour. In how many hours will they be 25 miles apart?

Here $(3x)^2+(4x)^2=(25)^2$



(3) Common facts such as the number of square rods in an acre, the number of yards in a mile or lbs. in a ton etc. are not known. This makes a term wrong in the equation. An examiner assumes that certain facts from arithmetical tables are common knowledge.

A rectangular field is 60 rods longer than it is wide. Its area is 10 acres. What are its dimensions?

While this is a quadratic equation it illustrates the point. The equation is $x(x+60)=160\times 10$ etc.

(4) The next step is the actual solution of the equation in the problem. This part of the work is more or less mechanical. Errors come under the headings of carelessness, poor uses of signs, ignorance of correct method etc. The previous work on equations has its reflection here. This is not the part of problem-solving which causes most difficulty. It may be well in this connection to note that pupils often think they are finished when they have found the value of x. *All* the information asked for in the question should be given.

Such are the difficulties facing the student who tries problem-solving. Many of these are more apparent than real.

In conclusion we append a plan which has been tried in the classroom and found very useful when the section on problems comes around again in the year's work. It is adhered to faithfully until it becomes quite mechanical.

Rules for problem-solving (one unknown).

(a) Read the question carefully so as to understand every part. (b) List the facts. (c) Represent the unknown by some letter x. Usually the unknown is really the one asked for in the question. If it is not, then let the unknown be that number to which the other facts are most closely related. (d) Represent the listed facts *algebraically*. (e) Group the facts around the unknown in such a way as to form an equation, checking to see that no *implied* fact has been omitted. (f) Solve the resulting equation for the unknown letter rejecting any value which does not suit the problem. (g) Give *all* the information asked for in the question.

Random Notes on Geometry Teaching

By HARRY C. BARBER

Note 3

The Introduction

THE first thing to do in the study of demonstrative geometry is to understand what demonstration is and why we demonstrate. If this is not done at the outset, the pupil becomes involved in misconceptions, doubts, and troublesome questions. "Why do we need to prove what we can see?" "What is the good of proving that?"

This understanding can be acquired without long labored introductions to demonstration and without verbal explanations of demonstration; it cannot be acquired by means of mere model demonstrations. Experience shows that it can be acquired by means of a brief pupil discussion of two topics.

First: Geometry is a game of, If you know. It asks, If you know *A* can you be certain of *B*. A score or two of questions in this form, some from geometry and some from homely matters of everyday experience, make this "game" completely clear to the pupil at once. He can understand that this is the game that geometry plays, and he can see that such reasoning is interesting and useful.

The second question is, *How do we know?* The pupil finds that he knows his age on the *authority* of his parents or of his birth record; he knows by *experience* that the hot stove will burn him; he *reasons* that the cat has eaten the canary when he finds the canary missing and yellow feathers on the cat's nose. Recognition of these three means of acquiring knowledge enables him to place reasoning, which is the method of geometry, in its proper relation to the other two methods.

The facing of these two ideas at the very outset, and occasional later reference to them is all that is necessary for the pupil's complete understanding of what demonstration is and why he is going to use it. This introduction completely eliminates the troublesome questions and other similar difficulties which arise when the pupil is not satisfactorily oriented at the outset.

The second thing for the introduction to do is to give some familiarity with the subject matter. This is often put first but that seems a mistake because the pupil is already more or less familiar with triangles, rectangles, circles, etc. And his point of view while he increases this familiarity ought to be the point of view which we are discussing. He ought to know that he is getting acquainted with the facts and the figures so that he can begin to reason about them. The value which he derives from the introductory drawings and the other devices for getting acquainted with the figures, will be doubled if he knows why he needs this familiarity and what use he is going to make of the figures. While he is doing the preliminary work he ought to know for what it prepares him.

In acquiring familiarity with the figures of geometry there can be no substitute for a certain amount of hand work, yet the formulation of one's own definitions gives just as precise a knowledge of the figures and an even better preparation for the intellectual activities called for in the demonstrations which are soon to come. Practice in formulating definitions may well be substituted for any excessive amount of drawing at the outset. Of course we ought to get more than we sometimes do out of the definitions we need in our argument.

In addition to certain definitions, mostly self made, the introduction ought to include a few assumptions,—a very few—, just enough to give a flavor; and some mention of the fact that there are terms which we do not attempt to define, as for example *point* and *line*.

The measurement aim of geometry (See

Note 2) should be mentioned in the introduction because it is the third essential idea. We *reason* about *geometric figures* with the aim of proving the relations needed for *measuring* them.

And finally we ought not to delay too long the mention of the approximate quality of all measurement. It can easily be made evident to the pupil and it is the best contrast with, and hence introduction to, the ideal figures about which geometry reasons.

◆ THE ART OF TEACHING ◆

A Christmas Project

By Muriel Batz, *Manitowoc, Wis.*

About a week before Christmas members of the Mathematics Club decided they would like a Christmas tree for their next meeting. Several boys volunteered to pick it out and supply a holder for it. They made a wise selection for they carried a well proportioned, nicely shaped balsam which displayed beyond a doubt their recent exposure to the topic of symmetry. With the tree they smuggled in an old Brer Rabbit gallon molasses pail filled with dampened sand. They promptly deposited their cylindrical container in one corner of the room. Then, one set about circumscribing Brer Rabbit with green wrapping paper, while the other braced the tree upright in the moistened sand.

Next, naturally, came the problem of ornaments. The club treasurer assured the members he could spare very little money. Whereupon, after considerable deliberation, they concluded trimming the tree would make an excellent project for the mathematics classes. This necessitated another shopping expedition for tinsel cord and sheets of silver, gold, and red tin foil paper, the kind which is used to wrap Christmas packages. These were cut into smaller pieces, some being one-fourth of the original size, others were one-eighth of the full size. A bit of diplomatic begging in the biology department and also in the library gave access respectively to thirty dissecting scissors and to a jar of paste. The latter was transferred to ten smaller vessels. Directions and patterns for making a rectangular parallelepiped, a cube, a cone, a cylinder, and a triangular pyramid were taken from "Mathematics for Everyday Use," by Stone and Mallory. It would have been better to have had these typed and mimeographed, but as time was limited, the instructions were given orally and the designs sketched on the blackboard.

When a class assembled the members were told to study the drawings on the board. During this interval, each pupil was given a scissors and a small section of tin foil paper. These were passed out, so that some received the one-eighth sizes of the red, some the one-eighth pieces of the silver, some the one-eighth sheets of the gold. Others received the one-fourth sizes of each of the three different colors of tin foil. Pots of paste were distributed about the room within convenient reach of every worker.

The members of the class were then instructed to make the largest solids possible out of their little portions of tin foil. They were advised to draw the pattern of the figure on the reverse side as that was plain and would show their pencil marks. From the left over scraps they were directed to cut out such surfaces as circles, rectangles, squares, trapezoids, triangles, semicircles or half moons, narrow strips or parallel lines, hexagons, and stars. Of course, their attention was called to the fact that the Christmas star is a five pointed one. It was necessary to have these plane figures duplicated, as they had to be pasted together; since, both sides would show when swinging on the tree.

The first row set to work making cubes, the second row boxes, the third row triangular pyramids, the fourth row cones, and the fifth row cylinders. As each pupil finished cutting out his figure, he pasted

it into shape, inserted a string, and then tied it on the tree. One inventive artist fastened a star to the apex of his cone. This he inverted over the crest of the tree for a top. Some attached their cylinders to the branches horizontally, others theirs perpendicularly. Some suspended their cones from the tips, others theirs with the vertices pointing downward. Some let their triangles, stars, and other plane figures dangle at the ends of long strings while others gave theirs less rope. Thus, the tree was trimmed, resplendent in all its glory of red, gold, and silver.

The members of the Mathematics Club were satisfied with the finished product and enjoyed it at their Christmas meeting. The members of the classes seemed to find doing the project an interesting lark. A few came after school to hang on extra figures they had finished during their free periods. The Geometrical Christmas Tree brought forth many and varied comments. But whether they were adverse or favorable, they all implied the same conclusion, that the ornaments were novel decorations.

There is the possibility of an entirely geometrical tree. The base can be a cylinder, the stem a very narrow rectangle, the main part of the tree several tiers of graduated trapezoids surmounted at the top by an isosceles triangle. If these are made from green cardboard, then mathematical figures of gold, red and silver tin foil can be pasted to them as ornaments. These make pretty spots of color against walls, or attractive window sketches.

Returning to the idea of the first tree comes the further speculation. Candy can be wrapped and inclosed within the solid figures. A rectangular parallelepiped will hold a nougat, a cube a caramel, a cylinder a chocolate covered Brazil nut, and a triangular pyramid a Hershey kiss. At a designated time on the dismissal day for the Christmas recess a raid on the Christmas tree can be permitted. For certainly, no fear or worry need be experienced over the destruction of the ornaments.

Meeting of the National Council of Teachers of Mathematics at Indianapolis, Indiana, December 29, 1937

9:00—Morning Session

Central Shrine—The World War Memorial Building

1. Address of Welcome—(15 minutes)—Walter G. Gingery, Principal, George Washington High School, Indianapolis, Indiana.
2. The Need of Cooperation between High School Teachers and College Teachers—(30 minutes)—Aubrey J. Kempner, President of the Mathematical Association of America, University of Colorado, Boulder, Colorado.
3. High School Teaching Values Derived from the Study of Higher Mathematics—(30 minutes)—J. O. Hassler, University of Oklahoma, Norman, Oklahoma.
4. What Shall We Do With These, Our Unfit?—(30 minutes)—Joseph Seidlin, Alfred University, Alfred, New York.
5. Mathematics Exhibits and the Opportunities Which They Present—(20 minutes)—Laura E. Christman, Nicholas Senn High School, Chicago, Illinois.

12:00—Luncheon

Spink-Arms Hotel, $1.00 per plate.

W. D. Reeve, Teachers College, Columbia University.

A Progress Report of the Work of the Joint Commission of the Mathematical Association of America, and the National Council of Teachers of Mathematics on the Places of Mathematics in the Secondary Schools.

2:15—Afternoon Session

Central Shrine—The World War Memorial Building.

1. Omissions, Enrichment and Improved Machinery Needed in All Three Types of Courses Found in Ninth Grade Mathematics—(30 minutes)—Edith L. Mossman, Berkeley, California.
2. The Role of Demonstrative Geometry in the Cultivation of Reflective Thought—(30 minutes)—Harold Fawcett, Columbus, Ohio.
3. Geometry, A Way of Thinking—(30 minutes)—H. C. Christofferson, Miami University, Oxford, Ohio.

Tentative Program of the Annual Meeting of the National Council of Teachers of Mathematics at the Hotel Traymore in Atlantic City, New Jersey, February 25 and 26, 1938

Thursday—8:00 p.m.

Board Meeting, Club Room, 10th floor.

Friday—9:00 a.m.

Board Meeting, Club Room, 10th floor.

Friday—2:30 p.m.

(a) Teacher Training Section, Rose Room, Mezzanine floor.

(b) Reports on the Status of Mathematics in the United States, Belvedere Room, 11th floor.

Friday—8:00 p.m.

General Meeting, Rose Room, Mezzanine floor.

Report of the Joint Commission.

Saturday—9:00 a.m.

(a) Arithmetic Program, Stratosphere Room, 8th floor.

(b) Senior High School Program, Submarine Grill.

11:45 a.m.

Discussion Luncheon, Belvedere Room, 11th floor (topics and leaders in November *Mathematics Teacher*).

2:15 p.m.

(a) Senior High School Program, Submarine Grill.

(b) Junior High School Program, Stratosphere Room, 8th floor.

3:45 p.m.

Business Meeting, Stratosphere Room, 8th floor, followed by meeting of New Officers and Board.

6:30 p.m.

Dinner, Belvedere Room, 11th floor.

IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

1. Boon, F. C. "The Solution of Equations by the Use of Proportional Differences." *The Mathematical Gazette*, 21:182–187. July, 1937.

The author states that a discussion of the construction of the difference columns in logarithm tables with graphical illustration would prepare a student for the methods described in the article "of an iterative process for the approximate solution of any equation."

Two problems are worked out in great detail to illustrate the method. The first is of the same type as the following: A goat is tethered to the circumference of a circular field; how long must his tether be (in terms of the radius of the field) if he can graze over half the field? The second problem solved is the well known one concerning a ladder leaning against a wall and passing over another.

2. Gorrell, George W. "Applications of Mathematics." *Bulletin of the Mathematics Section, Eastern Division, Colorado Education Association*, 2:3. October, 1937.

A brief discussion of the applications of various topics taught in the secondary courses in mathematics.

3. Langer, R. E. "René Descartes." *The American Mathematical Monthly*, 44:495–512. October, 1937.

An interesting biographical study and evaluation of the great French philosopher and mathematician, written in commemoration of the tercentenary of the publication of the "Geometry."

4. Nordgaard, Martin A. "Notes on Thomas Fantet de Lagny." *National Mathematics Magazine*, 11:362–373. May, 1937.

De Lagny lived in the years 1660–1734. He was "one of the lesser lights in a century great with mathematical luminaries of the first magnitude. But his work was by no means commonplace and his efforts took a direction away from the ordinary and beaten paths. He was the first to derive formulas for tan mx and sec mx directly from the right angle triangle, as he was the first to set forth in clear form the periodicity of trigonometric functions. . . ." A detailed account is given of his life and work and of his relation to his illustrious contemporaries. A full page photograph and numerous references are included.

5. Quick, Theadora. "Suggestions for the Slow-Learning Group in Junior High School Mathematics." *Bulletin of the Mathematics Section, Eastern Division, Colorado Education Association*, 2:8–10. October, 1937.

In September 1935, the author was assigned to teach a group of slow learning pupils entering the Lake Junior High School, Denver, Colorado. Throughout the seventh and eighth grades, these pupils met with her for english and mathematics. Many different units of work and methods of procedure were tried with varying degrees of success. The article deals with a few of the units of work in mathematics which seemed to be most worthwhile for this group.

6. Read, Cecil B. "Mathematical Magic." *School Science and Mathematics*, 37:847. October, 1937.

The largest number which can be written with two figures is not 99 but 9^9.

The theory underlying the following mathematical game is also described: "The first person names some number not greater than nine; the second may then add to that some number not greater than nine; the first now adds to this sum some number not greater than nine; and so on. The first person to reach 100 wins."

7. Richert, D. H. "Concerning the Teaching of the Linear Equation." *National Mathematics Magazine*, 11:382–384. May, 1937.

The writer makes some interesting observations on the pedagogic possibilities of the linear equation. If a student should ask what phenomena follow straight line trends, he should be advised to make a list of such phenomena from his study of the sciences. "He would be surprised at the endless number of instances in which the relation between variable quantities can be expressed by means of a linear equation."

8. Rutt, Norman E. "The Sources of Euclid." *National Mathematics Magazine*, 11:374–381. May, 1937.

The author analyzes the various forces that helped to bring about the *Elements* and concludes that the geometry of Euclid is compounded from the ingredients of engineering, art and religion.

9. Scott, Erma. "A Lesson on Proving Exercises." *Bulletin of the Mathematics Section, Eastern Division, Colorado Education Association*, 2:4–5. October, 1937.

The lesson is described in the form of questions, answers and discussion between the teacher and the pupils. The need for proof and the geometric methods of proof are discussed, with the teacher carrying, perhaps, too much of the burden.

10. Seidlin, Joseph. Review of "A Report of the Mathematics Committee of the California Junior College Association." *National Mathematics Magazine*, 11:385–389. May, 1937.

"A committee of junior college mathematics instructors appointed by the California State Department of Education made and published a survey of the mathematics courses taught in the public junior colleges of California. The report is an exhaustive document for which the committee deserves recommendation as well as honorable mention." The writer incorporates some of the more important tables, conclusions and recommendations in his review and makes some illuminating comments on them.

11. Shuler, Eucebia. "Application of Professional Treatment to Logarithms." *School Science and Mathematics*, 37:792–794. October, 1937.

This article constitutes part III of the series, "The Professional Treatment of Freshman Mathematics in Teachers Colleges." The topic of logarithms is treated under the following headings:

a. The history of the logarithms and the slide rule.

b. The history of the teaching of logarithms.

c. The theory of logarithms.

d. The graph of the logarithmic function.

e. The use of logarithms.

An extensive bibliography of thirty-five items is included.

12. Tate, M. W. "Teaching the Subtraction of Signed Numbers." *School Science and Mathematics*, 37:837–839. October, 1937.

It would take more than the space available to describe the proposed method of teaching the subtraction of signed numbers and to point out its pedagogic shortcomings. The following quotation will be sufficient evidence that it is *mathematically* false, as well:

"In summary: The method described above results in treating subtraction as a two step process: 1. Finding the absolute difference between the numbers. 2. Determining the sign of that difference by the reasonable principles that (a) when a smaller quantity is subtracted from a larger there is a favorable balance, a positive remainder and (b) when a larger quantity is subtracted from a smaller there is a deficit, a negative remainder." Evidently the above rules are not applicable to problems involving the difference of two algebraic numbers that have unlike signs.

13. Temple, G. "The Theory of Complex Numbers." *The Mathematical Gazette*, 21:220–225. July, 1937

The object of this article is "to review various theories of complex numbers and to scrutinize them from the standpoint of the teacher engaged in initiating his pupils into the subject." The following topics are discussed:

a. Problems raised by the introduction of complex numbers.

b. Complex numbers as operators.

c. Complex numbers as vectors.

d. Illustrations of the operational character of complex numbers.

14. Thielman, H. P. "A Generalization of Trigonometry." *National Mathematics Magazine*, 11:349–351. May, 1937.

Most students of mathematics are familiar with the idea that there exist several kinds of geometries and algebras. It has not occurred to many that there are various types of trigonometries. In the article the author presents a generalized trigonometry, special cases of which are familiar to every college student of mathematics. An interesting feature is a generalization of De Moivre's Theorem.

15. Wagner, George. "The Mathematics Work-Shop in the Eight-year Experimental Study of the Progressive Education Association." *Bulletin of the Mathematics Section, Eastern Division, Colorado Education Association*, 2:11–12. October, 1937.

"The Progressive Education Association had its second workshop this last summer at Sarah Lawrence College, Bronxville, New York. During the six-weeks period of the conference the mathematics group continued some of the work begun at the workshop at Ohio State University the summer of 1936 and worked on plans to be used in the classroom this coming year." The members of the group felt "that the teachers of mathematics were doing little toward teaching the pupil how to think in his everyday experiences. . . . The teachers were unanimous in believing that principles of logic and their transfer to life situations are of paramount importance to the pupil, while the acquisition of geometric facts and principles are of secondary importance." The major outcome of the workshop was "not in finished materials, but in the clearer realization of what the objectives in general education are, how mathematics can contribute to them, and the manner in which tests may be set up to see if pupils are applying their principles of logical thinking and reasoning."

NEWS NOTES

The third summer meeting of the National Council was held in the city of Detroit at the time of the Annual summer meeting of the N.E.A. The Local Committee with Miss Hildegarde Beck as Chairman, had made splendid arrangements before hand for taking care of the three-day session. On Monday, June 28 the following program was presented. The first session on arithmetic was as follows:

FIRST SESSION—ARITHMETIC

Monday, June 28—2:00 p.m.

Chapel of First Presbyterian Church
2918 Woodward Avenue

General Topic: Arithmetic Readiness in the Elementary Grades.

Presiding: H. C. Christofferson, Miami University, Oxford, Ohio.

1. The Transition from an Incidental to an Organized Program of Number Work—C. L. Thiele, Supervisor Exact Sciences, Detroit, Michigan.
2. Arithmetic Readiness and Curriculum Construction—Ben Sueltz, State Normal School, Cortland, New York.
3. The Development of Number Comprehension in the Primary Grades—R. L. Morton, Ohio University, Athens, Ohio.
4. A General Educator Looks at Arithmetic Readiness—Clifford Woody, University of Michigan, Ann Arbor, Michigan.
5. A Critique on the Report of the Committee of Seven—W. A. Brownell, Duke University, Durham, North Carolina.

JOINT SESSION WITH THE DEPARTMENT OF SECONDARY EDUCATION

Monday, June 28—3:00 p.m.

Cass High School

General Topic: Function of Extra Curricular Activities in Mathematics.

Presiding: A. Brown Miller, Cleveland, Ohio.

1. Use of Enrichment Materials in High School Mathematics—Joseph G. Shuttlesworth, Summit High School, Summit, New Jersey.
2. Mathematics in Photography (Illustrated)—Acenith Stafford, Evanston Township High School. Evanston, Illinois.
3. Field Work in Mathematics—Edwin A. Beito, University of Wichita, Wichita, Kansas.
4. Extra Curricular Activities in Junior High School Mathematics—Rose Boggs, Hibberd Junior High School, Richmond, Indiana.
5. Discussion—Norman Anning, University of Michigan, Ann Arbor, Michigan.

SECOND SESSION

Tuesday, June 29—2:00 p.m.

Chapel of First Presbyterian Church
2918 Woodward Avenue

General Topic: Mathematics and Allied Subjects.

1. An Answer to the Question, "Why Should Students Study Mathematics?"—Harry C. Carver, Professor of Mathematical Statistics, University of Michigan, Ann Arbor, Michigan.
2. Some Theoretical and Practical Mathematics Required by the Automobile Industry—J. O. Almen, Head of Dynamics Engineering Section, General Motors, Detroit, Michigan.
3. From the Study of Mathematics to an Appreciation of the Fine Arts (Illustrated)—Georg Wolff, Oberstudien Direktor, Duesseldorf-Oberkassel, Germany.

Tuesday, June 29—4:00 p.m.

Meeting of official delegates and state representatives in the Chapel of the First Presbyterian Church.

DISCUSSION LUNCHEON

Wednesday, June 30—12:00 Noon

Detroit Room, Detroit-Leland Hotel

The Discussion Luncheon held at the Detroit-Leland Hotel was a very successful meeting. Approximately 150 attended this function seated at tables for 10 and at each table some item of interest to mathematics teachers was discussed by a leader. The Detroit Mathematics Club arranged to have a Detroit teacher at each table to act as host. Each guest was presented with a favor made by pupils in the Detroit Public schools and also a collection of booklets on mathematical topics presented by the Detroit Mathematics Club.

THIRD SESSION

Wednesday, June 30—2:30 p.m.

Chapel of First Presbyterian Church

General Topic: Mathematics in the Ninth Grade.

1. Predicting Success in Algebra—Henry Eddy, Assistant Principal, Denby High School, Detroit, Michigan.
2. The Relationship of Qualitative Mental Differences to Learning in Mathematics—Harry J. Baker, Director of the Psychological Clinic, Detroit Public Schools.
3. A Short Survey of Work Being Done in Ninth Grade Mathematics—John Everett, Western State Teachers College, Kalamazoo, Michigan.
4. Discussion.

Local arrangements were in charge of the following: Hildegarde Beck, Enos Porter, Clarence Leonard, Agnes Crow, Stanley Barnes, Thera Smiley, Duncan Pirie, Lorna Tremper, Clara Mueller, May Walsh, Verna Philbrick, Elizabeth Nagelkirk, Fred Mulder, Bernhart Pagel, Byron Chapel, Louise Beck, Irene Sauble and C. L. Thiele, assisted by other members of the Detroit Mathematics Club.

ATTENDANCE

Arizona

Bisbee
    *Shepherd, Irene
Phoenix
    *Wilkinson
Tucson
    *Gale, Laura O.
    Kessler, R. V.

Arkansas

Fort Smith
    Spearman, J. A.

California

Berkeley
    *Mossman, Edith
Fresno
    Cronbach, L. J.
Hemet
    Birdick, A. A.
Los Angeles
    *Newcomb, Adeline
Oakland
    Benner, W. A.
    *Hesse, Emma
San Francisco
    Boulware, Louise
    *Vernon, Edith
Sanger
    *Spearman, Ethel

Colorado

Denver
    *Fergus, Lewis
    Gallup, Ruth

District of Columbia

*Grubbs, E. H.
*Harrington, K.

Florida

Gainesville
    *Kokomoor, F. W.

Georgia

Dallas
    Matthews, Mattie

Illinois

Bloomington
    Ahlenius, Ruth
    Arnold, Mary
    Clendmen, Ruth
Champaign
    *Bamberger, Alvena
Chicago
    *Christman, L. E.
    *Ely, Elsie M.
    Horton, Dorothy
    *Johnson, J. T.
    *Mullen, Frances
    Peterson, Ruth
    *Martin, Dorothy
    Wheeler, W. H.
Cicero
    Kreider, D. B.
Clinton
    *Foote, Frances
Elgin
    *Peters, Mary A.
Elmhurst
    Settle, Ida L.
Evanston
    Arnold, Erma
    *Stafford, A. V.
Hillsboro
    Kober, Marion
Joliet
    Mayo, Edward
Macomb
    *Schreiber, Edwin W.
Maywood
    *Hildebrandt, Martha
Quincy
    Hansen, C. W.
Raymond
    Bielema, Martin
Urbana
    *Nelson, Agnes L.
    *Taylor, S. H.
Winnetka
    Gleason, Ronald

Indiana

Elkhart
    Scott, Myra
Huntington
    *Shipley, M. E.
Indianapolis
    Coleman, Ada M.
    McClure, Byrl
Muncie
    *Whitecraft, L. H.
Plymouth
    Carothers, Bertha
Richmond
    Boggs, Rose E.
South Bend
    *Clark, Bernice E.
    *Kitson, Mary
    *Thornton, Wilson

Iowa

Cedar Falls
    Disney, Rhea
Fort Dodge
    *Hach, Alice
Washington
    Mead, Harland

Kansas

Pittsburg
    Lesher, Louise
Topeka
    Flare, May
    *Weaver, Ruth
Wichita
    *Beito, Edwin A.
    *Canine, C. H.
    *Kelly, Mary

Michigan

Ann Arbor
    *Chipman, Hope
    Norman, Anning
    *Lindell, Selma A.
    McLouth, Olive
    Rahoe, Nellie
    Rich, Hazel
    Woody, Clifford
Bay City
    Easterly, Isabelle
Birmingham
    *Mergard, Lila S.
Calumet
    Stott, Edith M.
Capac
    Smith, Ione L.
Cedar
    Priester, Elsie
Cement City
    Gallen, Helen
Charlotte
    Brown, Kenneth

Detroit
Barbour Intermediate
*Kniffen, Claude L.
Rosenfeld, Marie
*Trainor, E. A.
Cadillac
Kammen, Mollie
Carstens
Hanning, Alice L.
Cass High
*Chapel, Byron
*Keal, Harry
*Leiphart, Narcena
*McCullagh, S. E.
*Mueller, Clara H.
*Sprinkle, R. W.
Central High
*Brewer, Mary A.
*Marsh, E. H.
*Martin, H. L.
*Tayler, Mildred
*Wattles, H. M.
Chadsey High
*Barnes, Stanley
*Porter, Enos H.
Clark School
Eizen, Dorothy
Clippert
Robinson, Henrietta
Condon Intermediate
Austin, Olive
*Hobbs, Adelia
*Itsell, Mary R.
Dept. of Instruction
*Thiele, C. L.
Denby High School
*Mulder, Fred J.
*Myll, Loranie
Pagel, Bernhard
*Shires, Chas. T.
DurFee Intermediate
*Crone, Laurel
*Thompson, Wesley
East Commerce H. S.
*Beck, Louise
*Dougherty, Evelyn
McKinney, Margaret
Eastern High
Drew, Percy
Henze, Paula
Edgewood
Woods, May
Estabrook
Williamson, Ruth
Franklin
Miller, Jessie
Peguske, Ruth
Wood, Alice
Goodale
McDonald, May
Guest Elementary
Falik, Florence
Weise, Edith
Hally
Aldhous, Beatrice
H. P. H. S.
*Haupert, Ruth
Hutchinson
Bodzin, Jay
Hutchins, Int.
*Dietzel, W. H.
*Foster, Ellen K.
*Gaston, Harriet
*Pepper, Margaret
*Philbrick, V. B.
*Schwarg, Vivian
Jackson Intermediate
Hayes, Ruby
*Monaghan, Agatha
Rennie, Margery
Jefferson Intermediate
*Cowan, Beatrice
Merkobrad, B. T.
*Smiley, Thera
*Tremper, Lorana
Keating
Metting, Elfrieda
Leslie
Frisbie, M. K.
MacCulloch
Tremper, Catherine
Mackenzie High School
*Covey, Blanche
*Doub, Arnold V.
Miller High
*Cameron, Laura
*Nagelkirk, M. E.
*Vokes, Edna
Monnier
Samuelson, Louise
Munger Int.
*Quick, Martha
McMichael Int.
*Beck, Hildegarde
*Riley, Lucile
Teneyck, W. D.
Neinas Int.
*McVittie, Grace M.
Nichols
Steger, Gertrude
Nolan Int.
*Brandenburg, A. R.
Channel, Jeannette
*McDonald, Kate
*Young, Jane
Northern High
*Elliott, Ruth
King, Ruth E.
*Teninga, Gertrude
*Walsh, May
Walkins, E. E.
Northwestern High Sch.
Hulbert, W. O.
Mathews, Florence
Snyder, Gladys
Younglove, Florence B.
Pasteur
Fraub, Flossie
Post Intermediate
Berg, Hilda
Bristow, Deborah
Redford High School
Grover, Eunice
*Kingsburg, Rosa
Quick, Gula
Russell
Himel, Amelie H.
Sherrard Int.
*Dobbs, Olive
*Fern, Martin
*McNeil, Marion
Southeastern High
Beyer, Odell
Coughlan, Nina
*Crow, Agnes
Edwards, W. H.
Green, Lura
Klein, Adele L.
Leonard, Clara
Lyon, Edna
Shimp, Hiram
Sullivan, Margaret
*MacHale, Kathleen
Tappan, Int.
Doughty, Lloyd
Gillard, Melvin
University of Detroit
Johnston, L. S.
Von Steuben
Parham, Eva
Welsh, Mildred
Coll. of Ed., Wayne Uni.
Reitz, Wm.
*Worden, Orpha E.
Western High School
Clifford, Mary K.
Kruke, Rhea
East Detroit
Morris, W. W.
Flint
Adams, Marjorie
*Bishop, Florence
Davis, Frances
Ellis, Amber
*Hastings, Marie
*Loss, Nellie
Schultz, Alice
*Sweet, Ethel
Terry, Blanche
Terry, Rose F.
*Walz, Gretchen
Ferndale
Ford, C. C.
Harrison, Gerald
Kienbaum, Ervin
Maxwell, Olive
Fremont
*Bender, Irma
Grand Rapids
Bradford, Melvina
DeBorst, Janet
Dockeray, Eva
*Evangeline, Morrisey
Van Wicklin, Edith
*Wilson, Augeline
Grosse Pointe
Jackson, Humphrey

*Junge, Paul
Rich, Ruth
Hamtramck
Byrd, Helen
McCarthy, Margaret
*Miller, Helen
Highland Park
*Bywater, Celia
*Kirkendall, Geo.
Kirkendall, Rachel
Pendall, Florence
Holland
*Parkyn, Hannah
Jackson
*Field, Florence
Kalamazoo
*Everett, John P.
Hart, J. L., Jr.
Lambertville
Focge, Dorothy
Lansing
*Goodhue, Florence A.
Kyes, Neenah
*Seymour, Mildred
Sheldon, Ima
Lincoln Park
Henderson, Jessie L.
Melvindale
Priest, Almyra
Monroe
Wagner, Martha
Mt. Pleasant
Heilbronn, Edna
Knight, Thelma
*Montague, Josephine
Northport
Lautner, Sylvia M.
Otsego
Reid, N. P.
Pontiac
*Fox, Sadie
Hallenbeck, Ora
Steward, Margaret
River Range
Wheater, Helen
Royal Oak
Henry, Eleanor
Saginaw
*Mertz, Marie
Rabe, Marion
St. Johns
Graham, Esther
Sault Ste. Marie
Dobie, Alice
*Dow, Ethel J.
Sturgis
*Volpel, M. C.
Traverse, City
*Lautner, Elsie A.
Wixom
Gibson, Ethel
Ypsilante
*Matteson, Jane L.
*Turner, Mabel

Missouri

Columbia
*Butler, Charles
Calloway, Mrs. T. T.
Kansas City
*Young, Mary C.

Montana

Messorila
*Clark, Gertrude

Nebraska

Kearney
*Phalen, Eva

New Jersey

Summit
*Shuttlesworth, J. G.

New Mexico

Roswell
Harp, Ernest L.

New York

Cortland
*Sueltz, Ben A.
New Paltz
Higgins, Stella
New Rochelle
*Carroll, L. G.
New York
*Reeve, Wm. D.
*Williamson, R. D.
Valley Stream
*Henry, Etta M.

North Dakota

Fargo
Lyngstad, Anna

Ohio

Akron
Johnson, Lena L.
McKelvey, Augusta
Athens
*Benz, H. E.
*Morton, R. L.
Cincinnati
*Wrest, Alma
Cleveland
*Brown, Miller A.
Conneaut
Tobin, Marguerite
East Palestine
Blair, Waneta
Rowe, Julia
Rowe, Margaret
Genoa
*David, Fredrick
Huron
*Brown, Arnold
Lakewood
Palmer, Ethel
Lewistown
Thrust, M. V.
Oxford
*Christofferson, H. C.
*Rush, Mrs. C. H.
Selma
*Calvert, Helen
Shaker Heights
*Mille, F. B.
Steubenville
*Boyd, Margaret
Willoughby
*Wachtel, Irene

Oklahoma

Ponca City
*Greer, Lucile

Oregon

Portland
Beach, Agnes

Pennsylvania

Dubois
*Knarr, Malindac
Erie
Phalan, Thos.
Lewistown
*Cussman, Esther
Mifflintown
*Rearlick, Alice
Wellsboro
Peifer, John

Tennessee

Memphis
Morris, H. H.
*Wren, Frank L.

Texas

Dallas
Moore, Wilma V.
*St. Clair, Florence
Graham
*Sparks, Izetta
Houston
Popham, Thos. A.

Utah

Springville
Bearnson, Bertha
Provo
Johnson, W. F.
Thornton, J. W.

Washington

Seattle
Rushing, John
Van Orsdall, Otie P.

Spokane

*Bell, Kate
*Claussen, Christina

Wisconsin

Racine
Moore, Hugh
La Crosse
*Stokke, C. H.

*** Indicates membership in the National Council.**

ATTENDANCE BY STATES

Arizona	4	Michigan	196	Oklahoma	1
Arkansas	1	(Detroit)	117	Oregon	1
California	9	(Other Cities)	79	Pennsylvania	5
Colorado	2	Missouri	3	Tennessee	2
Dist. of Col.	2	Montana	1	Texas	4
Florida	1	Nebraska	1	Utah	3
Georgia	1	New Jersey	1	Washington	4
Illinois	27	New Mexico	1	Wisconsin	2
Indiana	10	New York	6		
Iowa	3	North Dakota	1	Total	318
Kansas	6	Ohio	20		

EDWIN W. SCHREIBER, *Secretary*

Professor E. V. Huntington of Harvard University delivered two lectures during July at the third annual conference of the Cowles Commission for Research in Economics at Colorado Springs.

Professor J. H. Van Vleck of Harvard has been appointed Visiting Professor at Princeton University for the first half of the present academic year.

Several Benjamin Peirce Instructorships at Harvard University for the year 1938–39 are open to men who have the degree of Ph.D. or its equivalent. Applications should be sent to the Chairman of the Division of Mathematics.

Mr. John A. Bath of the Mathematics Department of the Peru State Teachers College at Peru, Nebraska, writes:

"Of all the mathematics magazines published, *The Mathematics Teacher* stands out in a class by itself. The articles are stimulating and of real benefit to anyone who is interested in making advancements in the teaching of mathematics. No teacher of the subject should be without the inspiration and new trends which this publication brings to one. The editorial board is to be congratulated.

Best wishes for continued success in the publication of this fine periodical."

We appreciate these fine words and it is our hope that teachers of mathematics will support the work that the National Council is trying to do by telling other teachers about the magazine so that they too can share Mr. Bath's enthusiasm.

The Spring meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at the H. B. Lawrence School at Holyoke, Massachusetts, on Saturday, April 17, 1937.

MORNING SESSION

10:00—Social Gathering.

10:30—Welcome. Dr. Howard Conant, Principal, Holyoke High School.
"Notes from the Classroom"—Melvin J. Cook, Williston Academy.
"Marking Time"—Paul E. Werner, Tutoring School, Greenwich, Conn.
"Utilizing the Game Spirit in the Teaching of Elementary Algebra"—Miss Marguerite E. Lougee, F. A. Day, Jr. High School, Newtonville.

12:30—Luncheon.

AFTERNOON SESSION

1:45—Business Meeting—Election of Officers.
"Some Theorems about Numbers"—Neal H. McCoy, Smith College.
"Mathematics as it is, Mathematics as it ought to be"—George W. Creelman, Hotchkiss School.

Officers of the Connecticut Valley Section

Frederick L. Mockler, President, High School, Holyoke; Wm. Fitch Cheney, Jr., Vice-President, Connecticut State College; Miss Marie Litzinger, Secretary, Mount Holyoke College; Miss Agnes Hahn, Treasurer, Hartford High School, Hartford; Nelson A. Jackson, Director, Mount Hermon School; Raymond K. Morley, Director, Worcester Polytechnic Institute.

Past Presidents

Harry B. Marsh, '20; Percey F. Smith, '21; M. M. S. Moriarty, '22; Eleanor C. Doak, '23; Joe G. Estill, '24; Joshua I. Tracey, '25; *Lyon L. Norton, '26; *John W. Young, '27; Rolland R. Smith, '28; Harriet R. Cobb, '29; Melvin J. Cook, '30; Bancroft H. Brown, '31; Dorothy Wheeler, '32; H. W. Dadourian, '33; David D. Leib, '34; Nelson A. Jackson, '35.

* *Deceased.*

The Twenty-Third Annual Meeting of the Kansas Section of the Mathematics Association of America and the Thirty-Third Annual Meeting of the Kansas Association of Teachers of Mathematics was held at the Allis Hotel in Wichita on April 3, 1937. The following program was rendered.

FORENOON SESSION

10:00 a.m.—Ball Room

Joint Session—K. A. T. M. and M. A. A.

Presiding officer—R. G. Smith, Kansas State Teachers College, Pittsburg.

1. Address of Welcome—L. W. Mayberry, Superintendent of Schools, Wichita.
2. Response—Anna Marm, Bethany College, Lindsborg.
3. Report of the National Council of Mathematics Teachers in Chicago—U. G. Mitchell, University of Kansas, Lawrence.
4. Announcements—Mary Kelly, Wichita High School East.
5. Mathematics as a Universal and Permanent Element in Education—Monograph and Discussion—William Betz, Director of Mathematics, Rochester, New York.

NOON

Luncheon and Social Hour

Empire Room, Allis Hotel

Committee in charge—Lucy E. Hall, Wichita High School North; Cecil B. Read, University of Wichita.

AFTERNOON SESSION

1:30 p.m.—Ball Room

Joint Session—K. A. T. M. and M. A. A.

Presiding officer—Lorena Cassidy, Wichita High School East.

1. Business Session
   Announcements—M. Bird Weimar, Wichita High School North.
   Report of Secretary-Treasurer—Helen R. Garmann, Kansas State Teachers College, Emporia.
   Report of Nominating Committee and Election of Officers—Sara Belle Wasser, High School, Pratt.
   Report of Finance Committee—W. H. Hill, Kansas State Teachers College, Pittsburg.
2. Reorganization of Secondary Education (with Particular Reference to Mathematics)—William Betz.
   K. A. T. M. members remain in Ball Room for discussion on above monograph by Mr. Betz.
   M. A. A. members adjourn to Aviation Room for further program.

M. A. A. SESSION

Presiding officer—R. G. Smith.

1. Some Identities due to Cayley and their Vector Equivalents—A. S. Householder, Washburn College, Topeka.
2. Concerning Gamma Function Expansions—H. Van Engen, Kansas State College, Manhattan
3. An Unknown Property of Conics—G. W. Smith, University of Kansas, Lawrence.
4. Election of Officers.

Officers K. S. M. A. A.

Chairman, R. G. Smith, Kansas State Teachers College, Pittsburg; Vice-Chairman, W. G. Warnock, Fort Hays Kansas State College, Hays; Secretary-Treasurer, Lucy T. Dougherty, Kansas City Junior College.

Nominating Committee: W. T. Stratton, Kansas State College, Manhattan; Guy W. Smith, University of Kansas, Lawrence; O. J. Peterson, Kansas State Teachers College, Emporia.

Officers K. A. T. M.

President, Lorena E. Cassidy, Wichita High School East; Vice-President, Anna Marm, Bethany College, Lindsborg; Secretary-Treasurer, Helen R. Garman, Kansas State Teachers College, Emporia; Editor of the *Bulletin*, Ina E. Holroyd, Kansas State College, Manhattan;

Nominating Committee: Sara Belle Wasser, Pratt, Kansas; Ruth D. Payne, Wellington, Kansas; Esther Nicklin, Kansas City, Kansas.

The following officers of K. A. T. M. were elected for the year 1937–38: President—Miss Anna Marm, Bethany College, Lindsborg, Kansas; Vice-President—Miss Minnie Stewart, High School, Topeka, Kansas; Secretary-Treasurer—Miss Helen R. Garman, Teachers College, Emporia, Kansas; Editor of *Bulletin*—Miss Ina Holroyd, Kansas State Agricultural College, Manhattan, Kansas.

MARY KELLY

The Minneapolis Mathematics Club completed its active program for the year 1936–37 under the leadership of its president, Dr. L B. Kinney, head of the mathematics department of the University of Minnesota High School. Seven programs included the following:

1. "The Place of Mathematics in the Cirriculum" by Miss Prudence Cutright, Assistant Superintendent of Schools.

2. An open forum on "Elementary Algebra" led by Mr. Robert Drake of University High School.

3. "Mathematics in General" by Dr. Caldwell, Assistant Commissioner of Education.

4. An open forum on "Plane Geometry" led by Miss Edna Cockburn of Marshall High School.

5. Reports on the annual meeting in Chicago of the National Council of Mathematics by Miss Edith Woolsey of Sanford Junior High School and Dr. L. B. Kinney of University High School.

6 "Education and Mathematics" by Mr. Carroll R. Reed, Superintendent of Schools.

7. "The Place of Mathematics in Modern Education" by Dr. W. D. Reeve of Teachers College Columbia University.

May Moore, *Secretary*

A meeting of the mathematics teachers of the Bay Section of the California Teachers Association was held for Institute credit on Thursday, April 8th, at 4:00 o'clock at the Garfield Junior High School in Berkeley. It was one of the series of group meetings planned to get acquainted with the work of the mathematics teachers of the various cities in the section. The Berkeley teachers planned the subject: *Exhibits and Descriptions of Methods and plans found particularly interesting and valuable in the Berkeley Senior and Junior High Schools.* The program was as follows:

4:00—Informal half-hour questions, discussions, and examination of exhibits, and getting acquainted.

4:30—Five minute discussions by each of five mathematics teachers from Berkeley: Miss Alice Tyler, Senior High School, Berkeley; Mr. C. S. Cramer, McKinley High School, Berkeley; Miss Dora Ellen Martin, Burbank Junior High, Berkeley; Miss Minnie P. Mayne, Willard Junior High, Berkeley; Miss Edith Mossman, Garfield Junior High, Berkeley.

5:00—Questions and discussion of talks.

Mrs. Helen T. Hoefer, Advisory Chairman, presided.

The spring meeting of the Mathematics section was held in San Francisco on May 15th, 1937, in the Auditorium of the Health Building. Miss Mary McBride of San Francisco presided. Sixty members were present to hear Professor Sophia Levy of the Mathematics department of the University of California present a paper on Socializing Mathematics. Mrs. Helen T. Hoefer, the retiring Advisory chairman, gave a report on the seminar, Mathematics 260, given by Professor G. C. Evans and staff this spring. The attendance at the class was small. The topics of applications proved interesting and helpful. It is planned to continue the course next semester, changing the name to Coordination of Teaching of Mathematics. It will be under the direction of Professor Levy. As before, auditors will be welcome. Those who desire credit may register and receive two units. It will be given as a professional course instead of a seminar.

The business meeting followed with election of officers as follows: Advisory Chairman: Miss Bernice Cochran, Fremont High School, Oakland, California; East Bay Chairman: Miss Edith Mossman, Garfield Junior High School, Berkeley, California; East Bay Secretary: Miss Minnie P. Mayne, Willard Junior High School, Berkeley, California; West Bay Chairman: Miss Adeline Scandrett, Mission High School, San Francisco, California; West Bay Secretary: Miss Barbara K. Acheson, Mission High School, San Francisco, California.

It was voted to have a publicity committee. Miss McBride appointed Mr. A. L. McCarty of San Francisco Junior College as Chairman. Three teachers will be appointed from the East and West sections each, to represent the Junior College, Senior High School, and Junior High School fields. Articles on the teaching of mathematics will be submitted to various educational publications as well as to mathematical publications. Administrators, supervisors, and counselors should be informed of what is going on in the field of mathematics.

Professor Levy said that the medical department at the University of California has asked the mathematics department to give a year course in mathematics to combine Math. 3A, 3B, and 4A for pre-medical students at a time when they can take it. For the first time in years an elementary course in mathematics will be given in the afternoon when the students in medicine can take it.

Emma Hesse.

During the current school year marked progress has been made by the Cooperative Study of Secondary School Standards, which consists of representatives from the six regional associations of colleges and secondary schools in the United States. Two hundred secondary schools of a wide variety of types and sizes located in every one of the forty-eight states and the District of Columbia have been studied intensively in a number of ways since September 1936.

Each school has been visited for periods of two to ten days each by a committee of experienced educators who have made a detailed examination, analysis, and appraisal of the curriculum, pupil activity program, library service, guidance service, instruction, educational outcomes, staff, plant, and administration of each school after checking more than 1500 different items in these areas. They have paid particular attention to actual instruction, over three-fourths of the five thousand teachers having

been visited in their own classrooms, many of them being thus visited several times. The visiting committees have usually consisted of three or more men, at least one of whom has been a recognized educational leader in the state in which the school is located.

A group of ten test administrators has given a series of psychological, achievement, and social attitudes tests to 20,000 pupils in the cooperating schools. These tests were given in the early autumn, and alternative equivalent forms of them in the late spring, in order to measure progress during the school year.

Several other extensive studies have also been carried on in order to obtain other types of evaluation of the educational process and product in these schools. One of these involved securing reports on the subsequent collegiate success of over 16,000 graduates of the schools who later entered higher educational institutions. This information has been furnished by the registrars of the 1700 institutions of higher education which these graduates entered. Another significant study has been an investigation of the subsequent records and careers of almost 15,000 pupils in the 200 schools who did not go on with any further formal education after leaving high school.

Another factor in the appraisal of the schools has been the judgments of over 7000 parents of high school seniors concerning twelve different aspects of the school's influence on the lives of their sons and daughters. Still another factor is the judgment of approximately 20,000 pupils now enrolled in these schools.

Through tryout of these different methods of appraisal, the Cooperative Study will attempt to ascertain the validity and the relative importance of various ways of evaluating schools. In this way improved methods will be developed not only for accrediting secondary schools but also for stimulating these schools to become progressively better from year to year. These revised criteria, together with techniques for their application will, as a result of the Study, be available for the use of regional associations and State agencies in accrediting and improving secondary schools located in their respective territories.

It is expected that another year will be required before the data collected during the current year have been fully analyzed and resultant recommendations formulated. The five-year Study, which is now closing its fourth year of activity, is being financed by one of the educational foundations and by the six cooperating associations—the New England Association, the Middle States Association, the North Central Association, the Southern Association, the Northwest Association, and the Western Association. The total cost will be in the neighborhood of $160,000, in addition to contributed services on the part of a large number of voluntary assistants in all parts of the country.

The chairman of the general committee of twenty-one members in charge of the Study is Dr. G. E. Carrothers of the University of Michigan; the chairman of the Executive Committee of nine members is Dr. E. D. Grizzell of the University of Pennsylvania; the secretary of both committees is Mr. C. A. Jessen of the United States Office of Education; the coordinator of the Study, in charge of the Executive Office at 744 Jackson Place, Washington, D. C., is Dr. Walter C. Eells, of Stanford University.

Dr. Ben Sueltz, Professor of Mathematics at the State Normal and Training School of Cortland, New York, and Chairman of the State Syllabus Committee in elementary mathematics writes.

"The National Council of Teachers of Mathematics is to be congratulated on the publication of 'Numbers and Numerals.' Here is a monograph by two distinguished scholars who have written the story of numbers in simple and interesting language. I can think of no better way for a teacher in an elementary or secondary school to invest a quarter than in owning this book."

Waldemar Kaempffert, Science Editor of the *New York Times*, writes, "I hope that you will carry out that plan of publishing booklets like the one on 'Numbers and Numerals.' Inexpensive publications of this kind probably account for the higher cultural level of Germany as compared with the United States. If I can help you in any way to call attention to pamphlets still to come, please count upon me."

The women's Mathematics Club of Chicago and Vicinity held its last meeting of the year on May 15 in the Green Room at Mandel Brothers. The officers for the coming year were elected as follows: President—Dorothy Martin, Bloom Township High School; Vice-President—Ida D. Fogelson, Bowen High School; Secretary—Helen White, Englewood High School; Treasurer—Mary Jane Hartman, Oak Park High School; Editor—Ionia J. Rehm, Englewood High School; Chairman Program Committee—Marie Graff, Englewood High School.

After the election, the subject under discussion was "Mathematics Made to Order." The topics presented were:

1. Mathematics in Home Finance—Lenore King, Flower High School.
2. Mathematics in Economics 4A,—Martha Butler, Schurz High School.

3. Optional Mathematics Topics and Marks for Same—Clara Larson, Schurz High School.
4. A Mathematics Newspaper—Mrs. Mullen, Fenger High School.
5. New Mathematics Courses—Frances Hubler, Tilden Tech High School.

The Women's Mathematics Club wishes to express their thanks and appreciation for your cooperation during the past year.

IDA D. FOGELSON

Dr. Earle R. Hedrick, Professor of Mathematics at the University of California at Los Angeles, was recently named Vice President and Provost of the University. President Sproul of the University of California in speaking of Dr. Hedrick said,

"In my selection of Dr. Hedrick, I have chosen a man who is a sterling administrator and whose academic work is the very best. With his assistance, I shall have splendid reinforcement in the never-ending campaign to keep the University at the top in academic and cultural rating, and to make it adequate in offering the highest educational opportunities to the young men and young women of California."

Dr. Hedrick is well known to the members of the National Council and they will all be gratified to hear of this deserved promotion for him.

Mr. L. Denzil Keighley of Denver, Colorado, whose name appeared on the Official Primary Ballot of the National Council of Teachers of Mathematics on page 341 of the November issue of *The Mathematics Teacher* passed away recently according to word from Colorado.

One week after the November issue of *The Mathematics Teacher* appeared, forty reservations had been made for the Discussion Luncheon. If you wish a reservation at one of the tables announced (on page 340, November, *Mathematics Teacher*) please do not delay. Ten people, and no more, will be seated at each table.

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